α_{min}'s and α_{IV}'s is small. Although $G(r)$ oscillates and samples each orbital differently its effect in the calculation of the total exchange energy is very similar to that of α_{min}. This comparison between the factors multiplying the $\rho^{1/3}(r)$ exchange is possible because the charge densities determined from the $X_{\alpha\beta}$ and X_α method do not differ significantly. The calculated $G(r)$ was modified at large and small values of r in the same manner as in Herman's work.[1]

The results of many energy-band calculations have been shown to be sensitive to the exchange potential, and Slater's scheme, which is guided by first-principles arguments, seems more appealing than empirically determining α (or β) in a solid. However, the energy-band calculation of Cu by Snow[6] seems to indicate that the $\alpha = 0.83$ results are in much better agreement with experimental results from photoemission data, magnetoacoustic studies, and de Haas–van Alphen work than is $\alpha = 0.67$. Since $\alpha = 0.721$ is the value one would use in Slater's scheme, it is not clear that it will give the best results in some crystals. An energy-band study of Cu based on Herman's $X_{\alpha\beta}$ scheme, with $\beta = 0.0040$, is being undertaken to determine its merits.

The author would like to thank the following people in this Laboratory for their assistance: E. Kmetko for interesting discussions on the problem and for providing the results of his X_α calculations; J. H. Wood for the use of his computer program and for his interest in the problem; F. W. Schonfeld for encouragement; and W. Miner for reading the manuscript.

*Work performed under auspices of the U.S. Atomic Energy Commission.

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PHYSICAL REVIEW B VOLUME 2, NUMBER 6 15 SEPTEMBER 1970

Open-Orbit Resonances and Magnetic Field Dependence of the Ultrasonic Attenuation of Shear Waves in Magnesium†

R. V. Kollarits and J. Trivisonno
Physics Department, John Carroll University, Cleveland, Ohio 44118

and

R. W. Stark
The James Franck Institute and the Department of Physics, University of Chicago, Chicago, Illinois
(Received 20 February 1970)

The open-orbit resonances in the ultrasonic attenuation of shear waves in magnesium have been studied. The ultrasonic open-orbit resonances have been observed for magnetic fields applied in the basal plane for fields as high as 1100 G. The existence of the open-orbit resonance provides direct evidence for the presence of a spin-orbit–induced energy gap in the *AHL* Brillouin-zone plane. The period of the open orbit is in excellent agreement with the Brillouin-zone dimension in the [0001] direction. The effects of magnetic breakdown are observed to be of importance in fields of about 1 kG.

INTRODUCTION

Recent magnetoacoustic attenuation[1] and de Haas–van Alphen[2] experiments have led to a quantitatively accurate understanding of the electronic band structure[3] of magnesium. The nonlocal-band-structure calculation reported in Ref. 3 provided for the experimental data a detailed description whose accuracy was limited only by a calculational truncation error of about 1.5×10^{-3} Ry. This, however, was not sufficient to provide any information about the magnitude of spin-orbit splitting energy gaps which Cohen and Falicov[4] and Falicov and Cohen[5] had previously estimated to be about 5×10^{-4} Ry. As a consequence, the effects of spin-orbit coupling were included in Ref. 3 only through the implicit use of the single-zone scheme as a basis for the description of the electronic band structure and Fermi surface. The most important result that this leads to is to change the connectivity of one sheet of the Fermi surface; it becomes topologically open in the [0001] direction in contrast with the topologically closed situation which occurs in the absence of spin-orbit splitting. The dynamical response of the conduction electrons to a magnetic field applied perpendicular to the [0001] direction leads to a band of open orbits in the former case but to only closed orbits in the latter case. In the very-low-field limit ($H \leq 100$ G) the open-orbit description must be valid. Galvanomagnetic measurements[6] have shown that as a result of magnetic breakdown only the closed-orbit description is valid for higher magnetic fields ($H \approx 10$ kG).

Since the probability for magnetic breakdown is given by $P = e^{-H_0/H}$ with $H_0 \propto V_g^2$, where in this particular case V_g is the magnitude of the energy gap induced by spin-orbit coupling,[7,8] one can, in fact, determine V_g if one knows the value of the critical magnetic field H_0 descriptive of the transition from the low-field open-orbit case to the higher-field closed-orbit case. Falicov and Cohen[5] predict that this transition will occur with $H_0 \approx 200$ G.

The purpose of this paper is to present the results of an investigation of these open orbits using the magnetoacoustic open-orbit resonance technique. The amplitude of the resonances are studied as a function of magnetic field strength and orientation; the results yield additional confirmation of the quantitative accuracy of the Fermi-surface model as well as a value for the magnitude of the spin-orbit energy gap.

THEORY OF MAGNETOACOUSTIC EFFECT

The magnetoacoustic technique provides a direct way of measuring the extremal Fermi-surface calipers.[9,10] The ultrasonic attenuation observed when propagating an ultrasonic wave through a crystal while applying a magnetic field $\vec{H}$ perpendicular to the wave vector $\vec{q}$ is oscillatory and periodic in $1/H$. The extremal caliper is proportional to the periods of these geometric oscillations which are due to electrons traversing closed orbits on the Fermi surface. The theory is valid for both longitudinal and shear waves. But for shear waves, the polarization direction $\vec{p}$ must be perpendicular to both $\vec{H}$ and $\vec{q}$ and only a single direction of the magnetic field can be employed in a given experimental run.

Ketterson and Stark,[1] using longitudinal waves, found that the geometric oscillations in magnesium occurred on a rapidly increasing background attenuation, which made observation of the ultrasonic pulse impractical at fields above a few hundred Gauss. For shear waves, however, the background attenuation is not so large: It is thus possible to observe oscillations to higher fields. Furthermore, the strength and number of the geometric oscillations increases with ql, where l is the mean free path of an electron. Since the velocity of a shear wave is only about one-half that of a longitudinal wave in magnesium, a given ql value can be obtained with shear waves at approximately half the frequency required for longitudinal waves with a corresponding decrease in the background attenuation.

In addition to the geometric oscillations in the attenuation, sharp resonances may occur, which can be attributed to electrons traversing periodic open orbits. The theory of open-orbit resonances in the ultrasonic attenuation has been given by Galkin, Kaner, and Korolyuk.[11] A peak in the attenuation occurs when the period of the open orbit is an integral multiple of the sound wavelength. For the resonance condition to exist, it is essential that the open orbit have a periodic curvature so that the energy absorbed from the sound wave will exhibit the same periodicity.[12]

The field at which the resononace occurs is

$$H_r^n = cq\hbar G/2\pi n e, \quad n = 1, 2, 3 \cdots \tag{1}$$

where G is the period of the open orbit in reciprocal space. The highest field resonance then occurs at

$$H_r^1 = cq\hbar G/2\pi e\,. \tag{2}$$

The actual shape of the resonance is quite complicated, but its dominant form is that of a Lorentzian, which is given as

$$\frac{\alpha_0(q)\,\gamma}{\gamma^2 + n^2(1 - H_r^n/H)^2}\,, \tag{3}$$

where $\gamma = \pi n/ql$ and $\alpha_0(q)$ is that portion of the zero-field attenuation due to electrons which are contributing to the open-orbit resonance. For $ql \gg 1$, $\alpha_0(q)$ is proportional to q and independent of l, i.e., $\alpha_0(q) = Aq$. The maximum amplitude of the resonance is then proportional to the product of the mean free path and the square of the ultrasonic frequency

$$\alpha_{max} = Aq^2 l/\pi\,. \tag{4}$$

It can also be shown that the half-width of the resonance is

$$\Delta H = 2\pi H_r^1/ql\,. \tag{5}$$

The open-orbit resonance can thus be used to measure the mean free path of the electrons.

The validity of this theory has been adequately demonstrated by the work of Gavenda and Deaton[13] and Deaton and Gavenda,[14] who studied in detail the open-orbit resonances in zinc and cadmium. In particular, they found that the amplitude of the resonance line, its shape, and its position in H were in good agreement with the predictions of Eqs. (3)–(5).

The essential quantity that determines the resonance line shape is the value of ql. From Eq. (2) it is apparent that if q were doubled, the center field of the resonance would also double. Equation (4) shows that the new resonance line should then have a maximum attenuation four times larger than before.

Any deviations from this predicted behavior can then be attributed to changes in the effective electron mean free path l that are dependent on the magnetic field strength. One can expect l to depend on H when magnetic breakdown occurs. The open-orbit investigation that we report here utilizes this behavior to determine the value of the spin-orbit–induced energy gap in magnesium.

The most significant aspect of the above discussion is the predicted behavior of the amplitude of the resonance with q in the presence of an energy gap which is undergoing magnetic breakdown. Such a gap exists between the monster and the cap at the Brillouin-zone boundary shown in Fig. 1. (For a detailed discussion of the identification and relationship of the various pieces of the magnesium Fermi surface, see Ref. 1.) It has been estimated by Falicov and Cohen that this energy gap should be broken down for magnetic fields as low as 200 G. Due to the low-breakdown field, there has been no previous direct evidence to indicate the existence of this gap. The galvanomagnetic study made by Stark, Eck, and Gordon was performed at fields above a few kG which made it impossible to observe the small spin-orbit gap. Indirect evidence for the existence of this gap was obtained by Stark and Ketterson, who observed geometric oscillations associated with multizone calipers on the monster. The multizone orbits, which were observed to 300 G, can exist only if magnetic-breakdown effects are insignificant. For fields above 300 G, the background attenuation was so large in this experiment that no ultrasonic echoes could be detected.

From Fig. 1 it is clear that the inside of the arms of the monster will support open orbits parallel to the [0001] direction as a result of the spin-orbit gap. In absence of magnetic breakdown, open-orbit resonances will be observed with shear waves for $\vec{p}$ parallel to [0001], $\vec{q}$ parallel to [10$\bar{1}$0], and $\vec{H}$ parallel to [11$\bar{2}$0], or $\vec{q}$ parallel to [11$\bar{2}$0], and $\vec{H}$ parallel to [10$\bar{1}$0]. The period of these open orbits in reciprocal space will be $\vec{G}_1$, where $\vec{G}_1$ is the reciprocal-lattice vector along [0001].

The fields at which the lowest-order ($n=1$) and largest-amplitude open-orbit resonances associated with $\vec{G}_1$ should occur for $\vec{q}$ in the basal plane

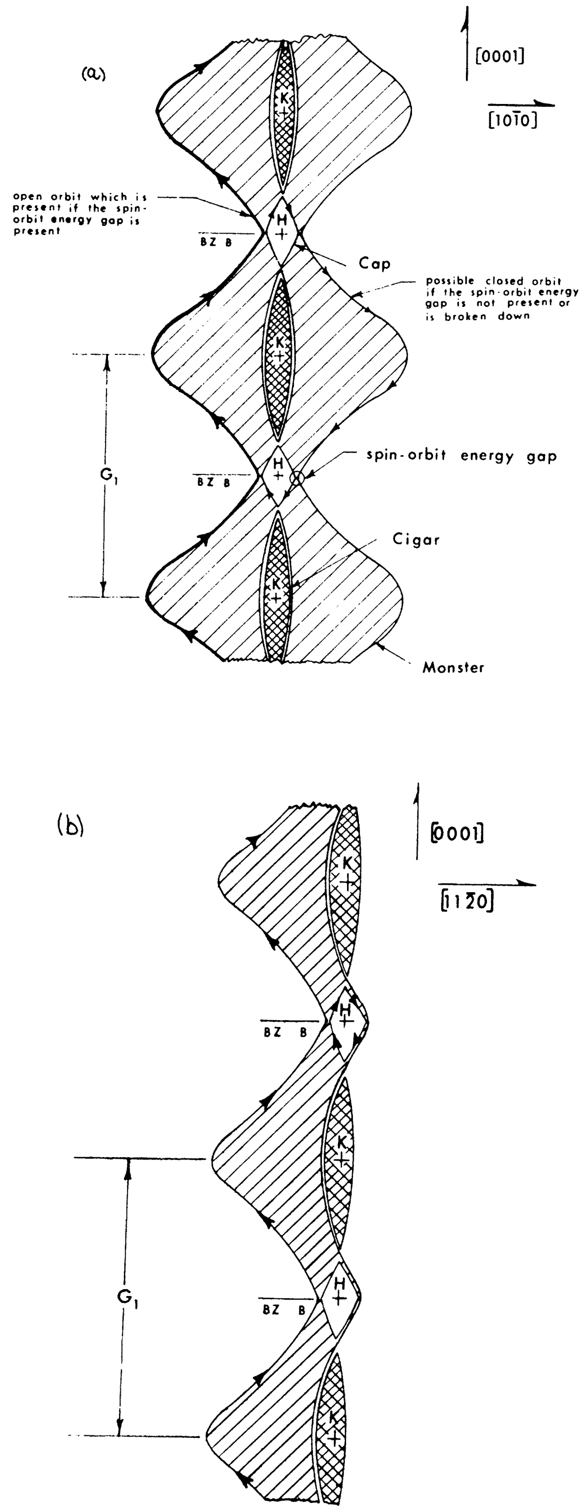

FIG. 1. Cross section of the Fermi surface of magnesium in the repeated zone scheme, where the Brillouin-zone boundary is indicated as BZB. (a) (11$\bar{2}$0) cross section. (b) (10$\bar{1}$0) cross section.

and p parallel to [0001] – assuming no magnetic breakdown across the spin-orbit gap – are given in Table I for experimentally realizable frequencies such as those used in this experiment.

If magnetic breakdown occurs, the increase of the resonance amplitude should be less than the frequency-squared dependence. Studies of the open-orbit resonance over a large frequency range can then provide information about the size of the gap. Measurements at low fields can give the Brillouin-zone dimension, a measure of the electron mean free path, and, as will be seen in a later section, information about the geometry of the Fermi surface.

TABLE I. Magnetic fields at which open-orbit resonances occur for the ultrasonic frequencies employed in this study.

f (MHz)	H_r^1 (G)
10	247
15	371
25	619
30	740
35	866
45	1113

EXPERIMENTAL TECHNIQUES

The experimental system used to observe the open-orbit resonances is shown in Fig. 2. For the longitudinal ultrasonic waves discussed in this paper, 10-MHz x-cut transducers were used in conjunction with a z-cut delay rod. For shear waves, either 5- or 10-MHz AC-cut transducers were used on an AC-cut delay rod. Since the AC-cut delay rod was piezoelectrically active, it was necessary to shield the receiving transducer from the end of the rod in order to prevent a capacitively coupled leakage pulse from obscuring the signal. This was accomplished by gold plating the delay rod and then grounding the rod. The transmitting $\frac{1}{2}$-in. coaxial plated AC-cut transducer was bonded to the delay rod with 6×10^4-cS (centistrokes) Dow Corning No. 200 silicon fluid. A $\frac{3}{8}$-in. transducer was used as the receiver. All magnesium to quartz bonds for shear waves were made with 3×10^4-cS Dow Corning No. 200 fluid.

In general, the linear i.f. amplifier was used to observe the geometric oscillations, whereas the logarithmic i.f. amplifier allows one to observe large changes in the pulse amplitude without having to change scales on the recording equipment. For the open-orbit resonances, it was also necessary to calibrate the y axis of the recorder plots in decibels. When the logarithmic i.f. amplifier is used, this calibration is nearly linear. The

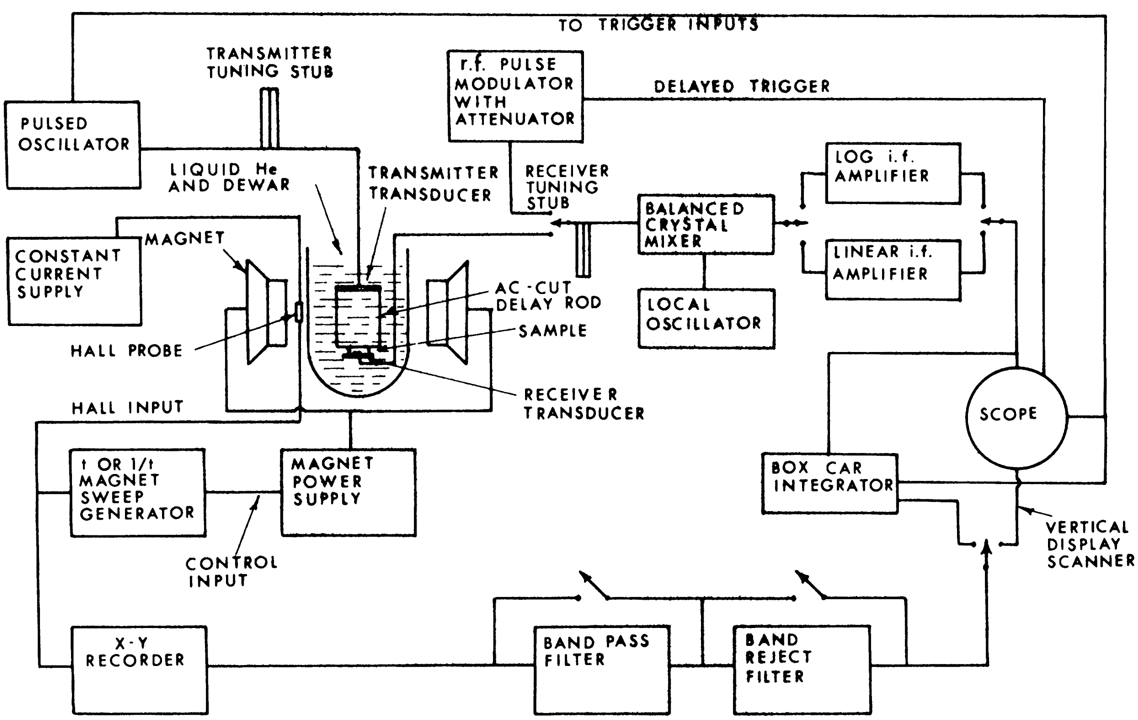

FIG. 2. Block diagram of experimental apparatus.

comparison pulse for the calibration was obtained by employing the delayed trigger pulse from a Tektronics 545 oscilloscope to externally pulse a Hewlett Packard 608D vhf signal generator. The carrier frequency of the signal generator was set to the value of the ultrasonic frequency, then fed into the receiver and its amplitude adjusted, using the calibrated piston attenuator until the output corresponded to that of the signal pulse. The piston attenuator was then used to calibrate the y axis on the recorder to within ± 0.3 dB. This 608D was also used to measure the frequency.

The x axis was calibrated in gauss using a 0.1% rotating-coil gauss meter whose calibration was checked frequently against a standard magnet.

The crystals used in the present experiments were the same used in the previous study of the magnetoacoustic effect in magnesium by Ketterson and Stark. The four crystals were all about 2 mm thick. Crystal 1 was oriented with $\vec{q}$ parallel to [0001], while crystals 2 and 3 were oriented with $\vec{q}$ parallel to $[10\bar{1}0]$ and crystal 4 with $\vec{q}$ parallel to $[11\bar{2}0]$.

In order to obtain bonds between the quartz and the magnesium, it was necessary to remove the oxide from the surfaces of the crystals. This was accomplished by lightly polishing the faces with jeweler's rouge and mineral oil.

The actual orientation of the crystals in the magnetic field was determined by several different methods. The orientation with respect to $\vec{H}$ was found to within $\pm 2°$ from the symmetry of the recorder plots of the geometric oscillations. It was also found for shear waves propagating in the basal plane that the high-field attenuation was a minimum when $\vec{H}$ was perpendicular to [0001], the direction of polarization of the wave. The increase in the high-field attenuation was not very sharp with angle, and consequently, this method only yielded the orientation to within $\pm 3°$. Knowing the orientation to this degree of accuracy, it was not too tedious to search in $\frac{1}{2}°$ steps for the open-orbit resonance. Once the resonance was found, the orientation could be determined to within $\pm 0.1°$.

The velocities of sound[15] in magnesium at 4.2°K which were used are the following: $V_{33} = 6.1444$, $V_{11} = 6.036$, $V_{44} = 3.24$, and $V' = 3.28$ (in units of 10^5 cm/sec).

RESULTS AND DISCUSSION

As a preliminary check of the system, geometric oscillations in the ultrasonic attenuation of longitudinal waves were studied in each of the crystals. Ultrasonic frequencies between 130 and 210 MHz were employed. The extremal calipers obtained from the periods of the oscillations are in good agreement with those obtained by Ketterson and Stark. As mentioned earlier, the magnesium crystals employed have a history of cycling between room temperature and liquid-helium temperatures which causes a deterioration in the electron mean free path. The oscillations were, however, still stronger and more numerous at 1.2 than at 4.2 °K, which is an indication that the attenuation is still phonon limited at 4.2 °K.

A study of the geometric oscillations with shear waves was also made. Measurements were made at frequencies as high as 135 MHz. The ql values associated with these 135-MHz shears are equivalent to those of 250-MHz longitudinal waves, but the background attenuation is lower and oscillations could be observed to fields as large as 800 G. The major advantage in using shear waves is to study oscillations and open-orbit resonances associated with orbits which may undergo magnetic breakdown since the lower background attenuation enables one to observe signals at higher magnetic fields than with longitudinal waves.

In addition to the geometric oscillations, which are due to closed orbits on the Fermi surface, ultrasonic open-orbit resonances were also observed. The resonances studied are due to open-orbit electrons traversing the arms of the monster parallel to [0001]. They were studied with shear waves in crystals 2 and 3 with $\vec{q}$ parallel to $[10\bar{1}0]$ and $\vec{p}$ parallel to [0001] and in crystal 4 with $\vec{q}$ parallel to $[11\bar{2}0]$ and $\vec{p}$ parallel to [0001]. No resonances were observed when $\vec{p}$ was parallel to the magnetic field direction, nor were they observed with longitudinal waves. Ultrasonic frequencies of 15, 25, 35, and 45 MHz were employed and resonances occurred at the fields listed in Table I. The existence of these resonances in magnesium is the first direct experimental evidence of the presence of the spin-orbit coupling induced energy gap between the monster and the cap at the Brillouin-zone boundary. The open-orbit resonances were quite similar to those observed by Gavenda *et al.* in cadmium and zinc. In these metals, however, magnetic-breakdown effects in the ultrasonic attenuation were not observed since their energy gaps are much larger than in magnesium.

The entire attenuation curve from $H = 0$ to $H = 10$ kG was taken at each ultrasonic frequency. These attenuation curves for frequencies between 15 and 45 MHz were all quite similar. Typical plots of pulse amplitude versus magnetic field are shown in Figs. 3 and 4.

Geometric oscillations occur at low magnetic fields and are associated with the lens. The open-orbit resonance occurs at the bottom of the last geometric extremum, which is a minimum in pulse height. The attenuation decreases and sat-

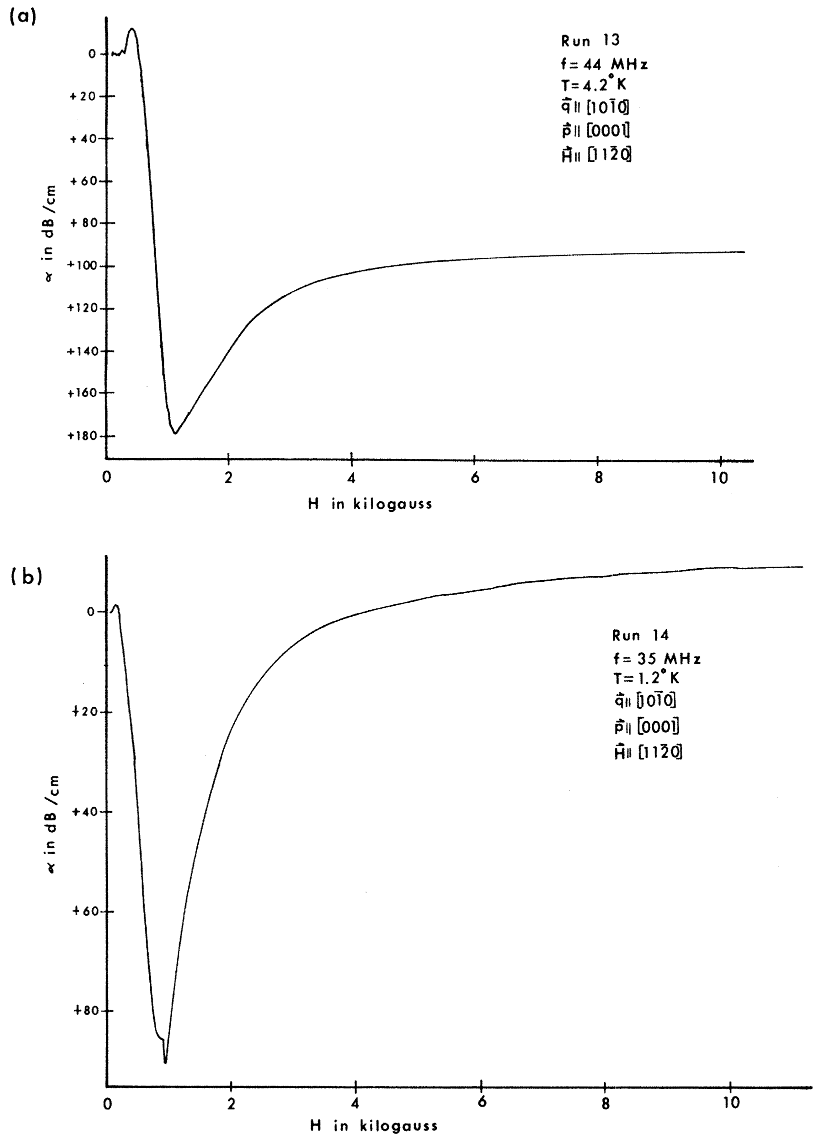

FIG. 3. Signal amplitude versus magnetic field for $q \parallel [10\bar{1}0]$.

urates at large fields. The general change in attenuation with frequency was found to be approximately proportional to the square of the ultrasonic frequency. At 45 MHz the attenuation of the last geometric extremum is 180 dB/cm below the zero-field attenuation. The expected change at 55 MHz is 270 dB/cm. This large attenuation yields a transmitted signal that is below the noise level of our system. Thus, we could not observe open-orbit resonances at frequencies higher than 45 MHz.

The open-orbit resonances in the ultrasonic attenuation can be observed only if a sufficiently large number of electrons are not tunnelling through the spin-orbit-induced energy gap; that is, magnetic breakdown is not occurring as the predominant mechanism for the removal of electrons from the open-orbit trajectory. If magnetic breakdown is not occurring, the amplitude of the resonance is proportional to the square of the ultrasonic frequency.

A plot of open-orbit resonance amplitude versus the square of the ultrasonic frequency is shown in Fig. 5 for data taken on crystal 4. The deviation from the frequency-squared law is apparent and is outside of experimental error. The fact that a large amplitude resonance is still present at 45 MHz indicates that magnetic breakdown is not complete in fields as high as 1100 G. For crystals 2 and 3, the open-orbit resonance was also observed at 1100 G, but the signal-to-noise ratio was poor. Measurements were made at 4.2 °K

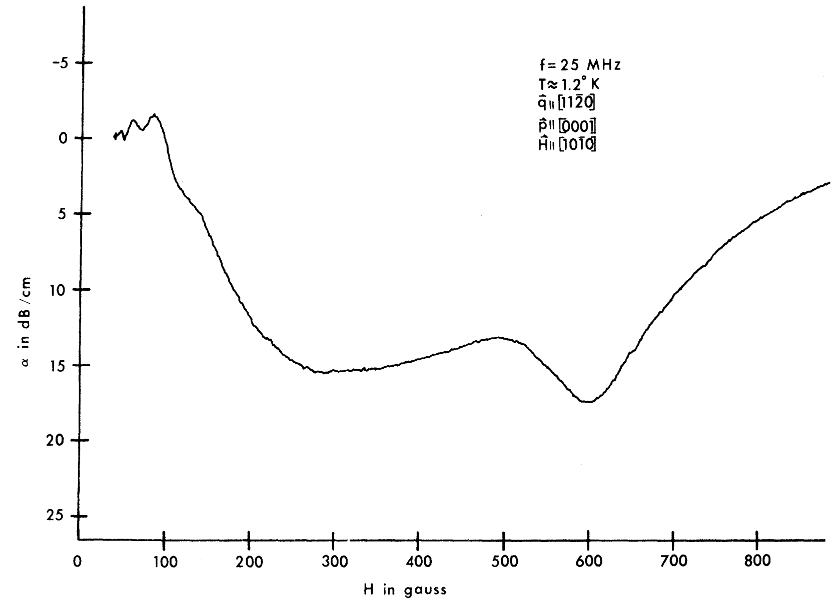

FIG. 4. Signal amplitude versus magnetic field for $q \parallel [11\bar{2}0]$.

to lower the background attenuation but the resonance amplitude could not be accurately measured.

Magnetic-breakdown effects account for the drop in the resonance amplitude at 1100 G. The resonance lines at 620 and 1100 G are compared in Fig. 6. The low-field line has a Lorentz shape. The high-field line has more structure than a simple Lorentz shape; this is probably due to the additional complexities that occur as magnetic breakdown adds extra kinks and twists to the open orbit. An exact explanation of the high-field line shape would require a detailed analysis of the theory for the particular orbit geometry existent in magnesium. However, the general shape of the high-field resonance line is so close to a Lorentz line shape that it is unlikely that significant error is made in analyzing the maximum attenuation.

Since the probability of an electron tunneling through the energy gap is proportional to $e^{-H_0/H}$

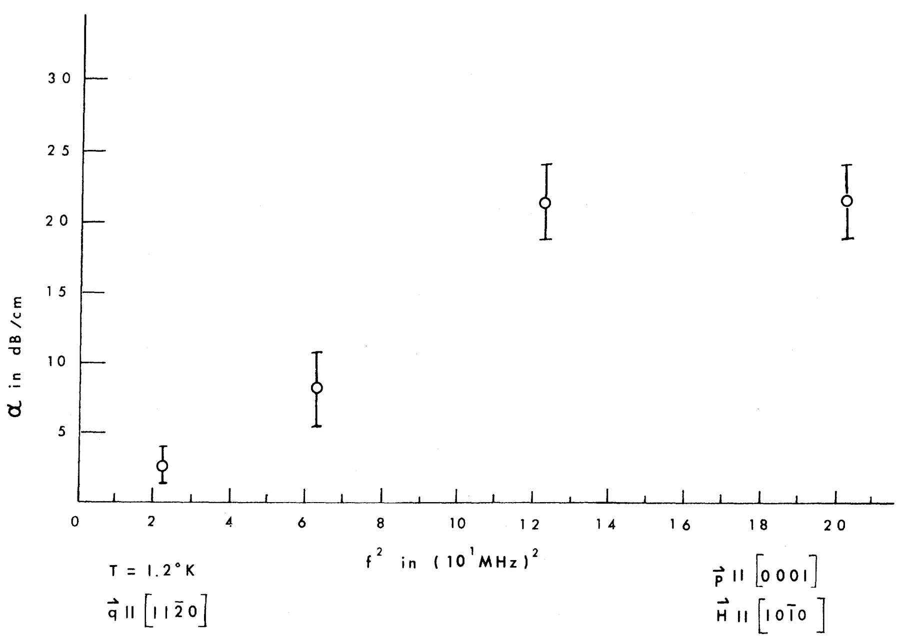

FIG. 5. Open-orbit resonance amplitude versus the square of the ultrasonic frequency.

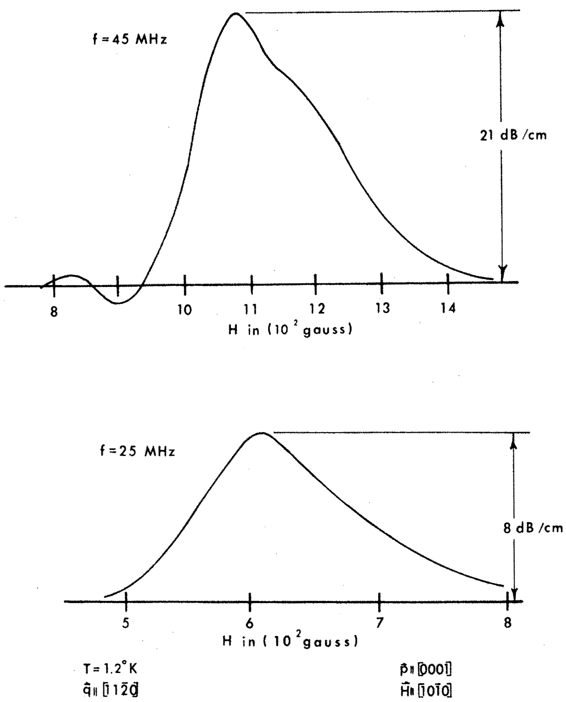

FIG. 6. Frequency dependence of open-orbit resonance lines. (The average background attenuation has been subtracted off.)

and since a strong open-orbit resonance is still present at 1100 G, H_0 must be considerably larger than the 200 G predicted by Cohen and Falicov. It is probably unwise to attempt to apply any simple model to the data shown in Fig. 5 in order to determine H_0. However, the significant deviation that occurs by 1100 G indicates that H_0 is of the order of 1 kG. Although this appears to be much larger than previously expected, it is compatible with our current understanding of the behavior of conduction electrons in magnesium. The formula for H_0 given in Ref. 8 is

$$H_0 = \tfrac{1}{4}\pi(cV_g^2/\hbar e V_x V_y), \qquad (6)$$

where V_g in energy gap and V_x and V_y are the two components of the electron (Fermi) velocity perpendicular to H, V_x being parallel to the Bragg planes and V_y perpendicular to them. For the particular geometry appropriate for the spin-orbit gap in magnesium this becomes

$$H_0 = \tfrac{1}{4}\pi \frac{cm_0(1+\delta)}{\hbar e} \frac{V_g^2}{E_F} \frac{K_F}{G(1-G^2/4K_F^2)^{1/2}}, \qquad (7)$$

where E_F is the Fermi energy, δ is the mass-enhancement factor, and the last term is an obvious geometry factor. V_g is taken to be 1.5 times larger than the appropriate atomic spin-orbit splitting since this is the factor which was found appropriate for zinc and cadmium. The actual values of the quantities as well as references to their sources are listed in Table II. The evaluation of Eq. (7) yields $H_0 = 900$ G, which is not inconsistent with the experimental data.

The dependence of the resonance amplitude on temperature is shown in Fig. 7. These plots were used to estimate the mean free path and to compare the quality of the various crystals employed in the present study. The variation of the resonance height and width with mean free path was similar to that reported by Gavenda *et al.* in cadmium and zinc.

The electron mean free path estimated from the half-width of the resonance line is approximately 2×10^{-1} cm for crystal 3 and 10^{-1} cm for crystal 4. Another, but less reliable, method of obtaining the mean free path is to observe the lowest field at which the geometric oscillation occurs. For the lens orbit the field was about 60 G. Since $\omega_c\tau$ must equal or be greater than 1 for oscillations to occur, a value of l can be estimated. The mean free paths determined in this manner are in agreement with those listed above. These large mean free paths indicate that the open-orbit electrons are traversing approximately six Brillouin zones in the lower magnetic fields used in this experiment and approximately 20 Brillouin zones in the higher fields.

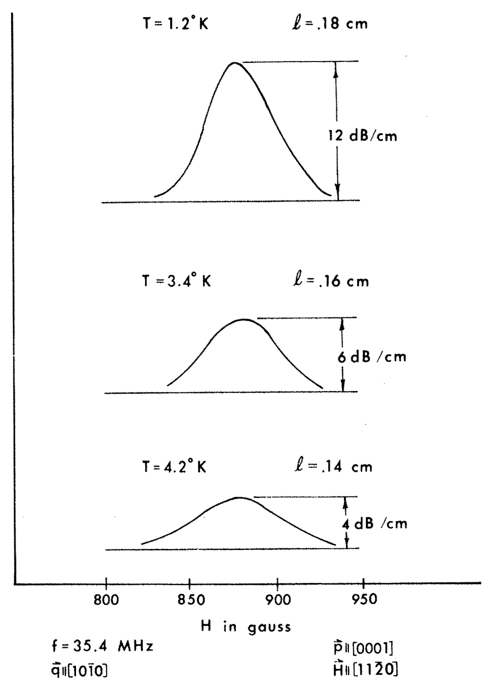

FIG. 7. Temperature dependence of the open-orbit resonance for $q \parallel [10\bar{1}0]$. (The background attenuation has been subtracted.)

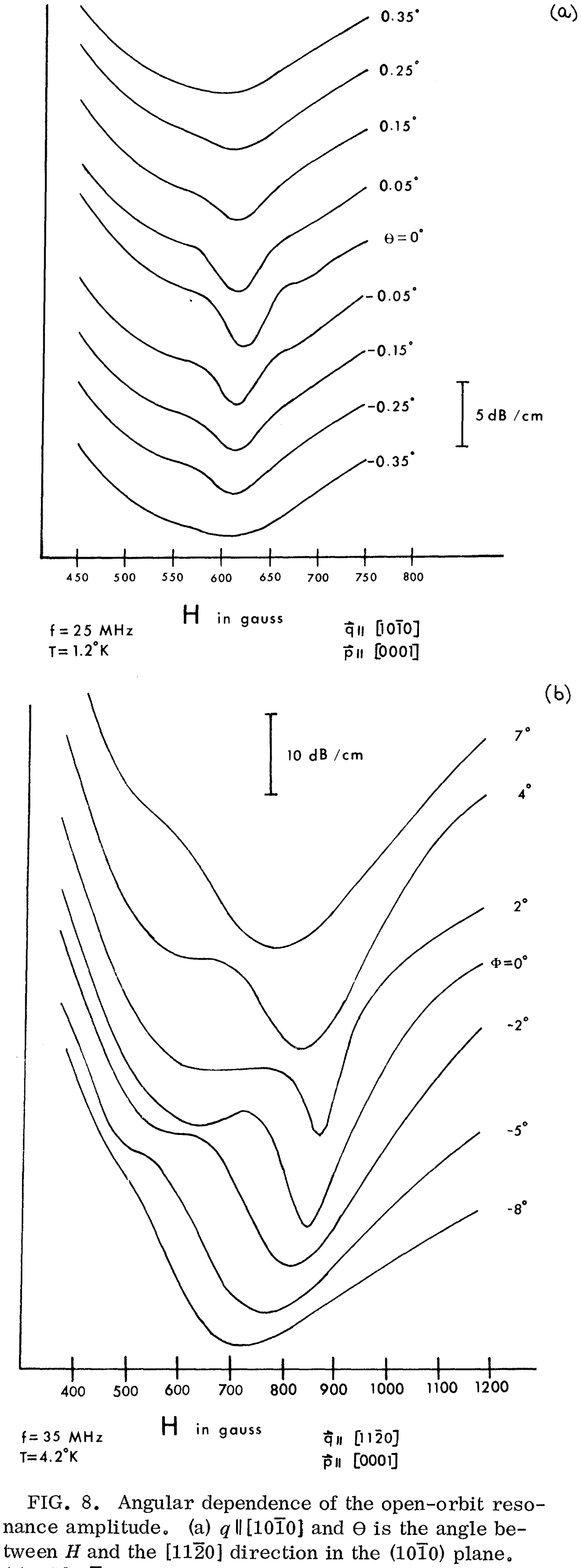

FIG. 8. Angular dependence of the open-orbit resonance amplitude. (a) $q \parallel [10\bar{1}0]$ and Θ is the angle between H and the $[11\bar{2}0]$ direction in the $(10\bar{1}0)$ plane. (b) $q \parallel [11\bar{2}0]$ and φ is the angle between H and the $[10\bar{1}0]$ direction in the $(11\bar{2}0)$ plane.

TABLE II. Values of parameters used in Eq. (7).

$k_f = 0.7275$ a.u.	Ref. 1
$G = 0.6424$ a.u.	Ref. 1
$E_F = 0.5292$ Ry	Ref. 1
$V_g = 5.6 \times 10^{-4}$ Ry	Refs. 16 and 17
$\delta = 0.26$	Ref. 3

The open-orbit resonance amplitude shows a strong angular dependence as the magnetic field is tilted out of the basal plane. This angular variation is shown in Fig. 8. The open-orbit resonance associated with $\vec{H}$ parallel to $[11\bar{2}0]$, the ΓK direction, is observed only over a very narrow angular range of approximately $\frac{1}{2}°$ as $\vec{H}$ is rotated in the $(10\bar{1}0)$ plane. The resonance associated with $\vec{H}$ parallel to $[10\bar{1}0]$, the ΓM direction, exists for nearly 6°, as $\vec{H}$ is rotated in the $(11\bar{2}0)$ plane. It is interesting that although the single orthogonalized plane-wave (SOPW) model of the Fermi surface of magnesium cannot explain this large angular anisotropy (a ratio of about 2 is expected with limiting angles of approximately 8° and 14° for $\vec{H}$ parallel to $[11\bar{2}0]$ and $[10\bar{1}0]$, respectively), it is, however, in good agreement with the Fermi-surface model obtained from band calculations. A projection of the monster onto the basal plane based on these calculations is shown in Fig. 9. When $\vec{H}$ is along

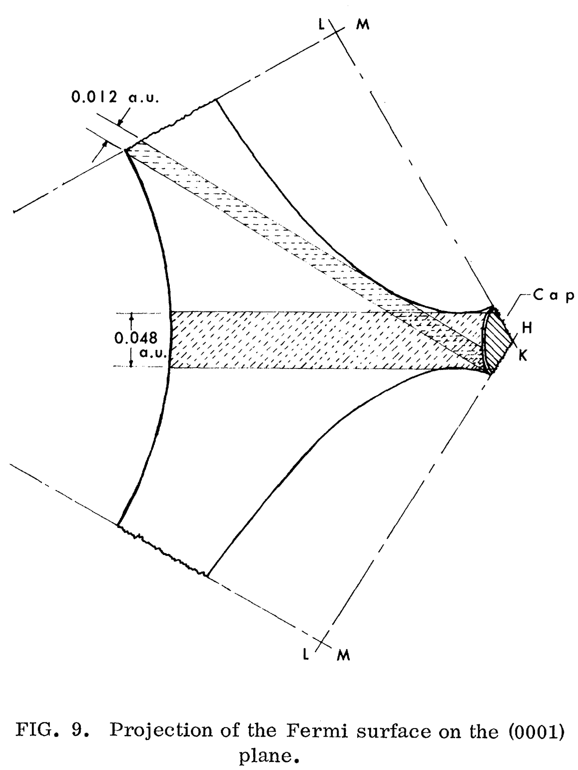

FIG. 9. Projection of the Fermi surface on the (0001) plane.

ΓM, the width of the band of open-orbit electrons is seen to be approximately 0.048 a.u. When H is along ΓK, the width is 0.012 a.u. Since the heights of the Brillouin zone are 0.64 a.u., the limiting angles (electron traversing one Brillouin zone) are approximately 1° and 4.3° for H along ΓK and ΓM, respectively. Detailed analysis from the band-structure model shows that the angles are $\frac{1}{2}°$ and 5°, respectively. Hence, the band calculations are in much better agreement with experimental results than the SOPW model and do show that the arms of the monster are highly constricted. The larger band of electrons, when $\vec{H}$ is parallel to ΓM, also accounts for the larger resonance amplitude observed in this direction, despite the fact that the crystal had a lower mean free path as determined from the resonance half-width measurements as shown in Figs. 6 and 7. For a given mean free path, the amplitude should be a factor of 4 greater when $\vec{H}$ is in the ΓM direction since the bandwidth is four times greater.

CONCLUSION

The existence of open-orbit resonances confirms the existence of an energy gap in the (0001) plane and the evidence of magnetic breakdown of the gap which is reported indicates that magnetic breakdown occurs at about 1 kG, which is somewhat higher than predicted by Falicov and Cohen. This high field is, however, compatible with our current understanding of the electronic band structures of magnesium, zinc, and cadmium. Also, the observed variation of open-orbit resonance amplitude with crystallographic direction and its angular variation as $\vec{H}$ is rotated out of the basal plane are in agreement with the band calculations and provides evidence for the constrictions in the monster arms predicted by these calculations.

ACKNOWLEDGMENTS

The authors wish to thank Professor W. L. Gordon for many stimulating discussions during the course of this investigation.

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†Work supported by AFOSR Grant No. 68-1479 to John Carroll University, Cleveland, Ohio, and Army Research Office (Durham) Grant to University of Chicago, Chicago, Ill.

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